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METHODS OF PREDICTING LAMINAR HEAT RATES ON HYPERSONIC VEHICLES

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E-260

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METHODS OF PREDICTING LAMINAR HEATING RATES ON HYPERSONIC VEHICLES

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SUMMARY

A summary of some of the simplest and best available laminar heat-transfer theories for flow in thermodynamic equilibrium is presented. In some cases the effects of frozen flow are included. Emphasis is placed on the proper methods of obtaining heating rates to hypersonic bodies, wings, and control surfaces. The effects of yaw and the determination of the inviscid flow field are also considered.

INTRODUCTION

The flight path of many hypersonic vehicles such as global gliders or reentering satellites is such that maximum heating will occur at very high altitudes. Since the Reynolds numbers will be quite low, the boundary layer is expected to remain laminar for some distance away from the leading edge. Although maximum heating will occur at high altitudes, the flow will still be in the continuum regime, and in many instances equilibrium dissociations will occur. Therefore, the design of high-altitude hypersonic vehicles will require a knowledge of the best available laminar heat-transfer theories for dissociated flow in thermodynamic equilibrium.

At present, the engineer desiring to estimate laminar heating rates must be familiar with many different theories and a great many references. This report combines many of the theories into one source and outlines some of the simplest available methods for calculating heating rates for equilibrium flow. All the theories presented apply to an isothermal wall only.

SYMBOLS

a sonic velocity

c_i mass fraction of i^{th} component

$\bar{c}_p$ — $\sum c_i \frac{dh_i}{dT}$

D — body diameter

D_i — diffusion coefficient

D_i^T — thermal diffusion coefficient

f — see eqs. (10)

g — H/H_e

H — total enthalpy

h — static enthalpy

h_D — average atomic dissociation energy times atomic mass fraction in external flow

h_i — enthalpy per unit mass of i^{th} component

h_i^o — heat evolved in formation of component i at 0^o R per unit mass

h_o — perfect-gas enthalpy per unit mass

$\bar{k}$ — frozen thermal conductivity

L_i — Lewis number, $D_i \rho \bar{c}_p / \bar{k}$

L_i^T — thermal Lewis number, $D_i^T \rho \bar{c}_p / \bar{k}$

l — $\rho\mu/\rho_{wo}\mu_{wo}$

M — Mach number

m_o — molecular weight of undissociated air

Nu — Nusselt number

$\overline{Pr}$ — Prandtl number, $\bar{k}/\mu\bar{c}_p$

p — pressure

q — heating rate

R — gas constant

E-260

R_N nose radius

R_x small radius of curvature at stagnation point of a three-dimensional body

R_z large radius of curvature at stagnation point of a three-dimensional body

Re_w Reynolds number, $\rho_w u_e x/\mu_w$

Re_x Reynolds number, $\rho_e u_e x/\mu_e$

r cylindrical radius of body

r_s blunt-body sonic coordinate, see fig. 2

s_i $c_i/c_{i,e}$

T temperature

u chordwise or longitudinal velocity component

V flight velocity

v spanwise velocity component

w velocity component parallel to shock

x longitudinal coordinate

x_n chordwise component normal to leading edge

x_s blunt-body sonic-point coordinate, see fig. 2

y normal coordinate

Z ratio of molecular weights, undissociated air to dissociated

α angle of attack or root angle of aerodynamic surface

α' effective flow deflection angle

β pressure-gradient parameter

γ ratio of specific heats

δ flow deflection angle

E-260

CN-1 back

η see eqs. (3)

Θ T/T_e

θ shock angle

Λ yaw angle

μ absolute viscosity

ν dynamic viscosity

ξ see eqs. (3)

ρ density

φ angular position

Subscripts:

as axisymmetric

aw adiabatic wall

e external flow outside boundary layer

eff effective conditions

w wall or surface value

wo stagnation wall value

η differentiation with respect to η

Λ quantity pertaining to yawed body

0 stagnation value behind normal shock

1 flight conditions, or conditions upstream of shock

2 conditions downstream of shock

2D two dimensional

3D three dimensional

E-260

Superscripts:

α see eq. (25)

n see eqs. (3)

* based on reference temperature

TRANSPORT OF HEAT

When the chemical reaction rates through a boundary layer are very large, the concentration of atoms within the boundary layer will be in thermochemical equilibrium. The temperature gradient through the boundary layer then uniquely determines the concentration gradient, and the enthalpy-temperature relation depends only on the pressure. Thus, the flow is considered to be in a state of thermodynamic equilibrium.

The local heat-transfer rate to a body surrounded by flow in thermodynamic equilibrium is determined by the sum of the heat transferred by conduction and the heat transferred by diffusion. In reference 1, the heat transfer to the wall is given by

$$q = \underbrace{\left(\frac{\bar{k}}{\bar{c}_p}\frac{\partial h}{\partial y}\right)_{y=0}}_{\text{Conduction}} + \underbrace{\left[\sum \rho\left(h_i - h_i^o\right)\left(D_i\frac{\partial c_i}{\partial y} + D_i^T\frac{c_i}{T}\frac{\partial T}{\partial y}\right)\right]_{y=0}}_{\text{Diffusion}} \tag{1}$$

The first term in equation (1) merely represents the heat that is conducted because of the temperature gradient normal to the wall. A second term, however, must also be added since at high temperatures a concentration gradient of atoms exists between the edge of the boundary layer and the wall. The second term in equation (1) represents the amount of recombination energy released at the wall due to the diffusion of atoms through the boundary layer.

By using the dimensionless variables suggested in reference 1, equation (1) becomes

$$q = \frac{r^n \rho_w \mu_w u_e H_e}{\sqrt{2\xi}\ \overline{Pr}}\left\{g_\eta + \sum \frac{c_{i,e}\left(h_i - h_i^o\right)}{H_e}\left[\left(L_i - 1\right)s_{i,\eta} + L_i^T s_i \frac{\Theta_\eta}{\Theta}\right]\right\}_{\eta=0} \tag{2}$$

where

$$\left.\begin{aligned} \eta &= \frac{u_e r^n}{\sqrt{2\xi}} \int_0^y \rho \, dy \\ \xi &= \int_0^x \rho_w \mu_w u_e r^{2n} \, dx \\ g &= H/H_e \\ \Theta &= T/T_e \\ s_i &= c_i/c_{i,e} \end{aligned}\right\} \tag{3}$$

E-260

and the ratio of the atomic diffusion coefficient to the thermal diffusivity is the Lewis number L_i, while the ratio of the thermal diffusion to the thermal diffusivity is represented as the thermal Lewis number L_i^T. For two-dimensional flow $n = 0$, while for axisymmetric flow $n = 1.0$. For the case in which no recombination occurs at the wall (concentration gradient vanishes) or for the special case $L_i = 1$ and the usual boundary-layer assumption $L_i^T = 0$, the summation term in equation (2) vanishes. Equation (2) then becomes

$$q = \frac{r^n \rho_w \mu_w u_e H_e}{\sqrt{2\xi}\ \overline{Pr}} \left(g_\eta\right)_{\eta=0} \tag{4}$$

If the following are defined,

$$Nu = \frac{q x \bar{c}_{p,w}}{\bar{k}_w (h_{aw} - h_w)} = \frac{q x \ \overline{Pr}}{\mu_w (h_{aw} - h_w)} \tag{5}$$

$$Re_w \equiv u_e x / \nu_w \tag{6}$$

the heat-transfer rate for the case of zero yaw can be written as

$$q \equiv \frac{Nu}{\sqrt{Re_w}} \frac{\sqrt{\rho_w \mu_w}}{\overline{Pr}} \sqrt{\frac{u_e}{x}} \left(h_{aw} - h_w\right) \tag{7}$$

where

$$h_{aw} \equiv h_e + \sqrt{\overline{Pr}} \frac{u_e^2}{2}$$

and the value of $Nu/\sqrt{Re_w}$ must be obtained from the boundary-layer solutions, while all other parameters needed in equation (7) are determined from the inviscid flow field and the wall temperature.

E-260

STAGNATION FLOW

Lewis Number of Unity

By neglecting thermal diffusion $\left(L_i^T = 0\right)$ and assuming a Lewis number equal to 1.0, the laminar boundary-layer equations for a dissociated gas at a two-dimensional or axisymmetric stagnation point from reference 2 are written as:

Momentum:

$$\left(l f_{\eta\eta}\right)_\eta + f f_{\eta\eta} + \beta\left(\frac{\rho_e}{\rho} - f_\eta^2\right) = 0 \tag{8}$$

Energy:

$$\left(l g_\eta/\text{Pr}\right)_\eta + f g_\eta = 0 \tag{9}$$

where

$$\left.\begin{aligned} l &= \rho\mu/\rho_{wo}\mu_{wo} \\ f &= \int_0^\eta f_\eta \, d\eta \\ f_\eta &= u/u_e \\ \beta &= \frac{\xi}{u_e}\frac{du_e}{d\xi} \end{aligned}\right\} \tag{10}$$

and g and η are defined by equations (3). The boundary conditions are

$$\left.\begin{aligned} &\eta = 0 \qquad f = f_\eta = 0 \qquad g = g_w = h_w/H_e \\ &\eta \to \infty \qquad f_\eta \to 1 \qquad g \to 1 \end{aligned}\right\} \tag{11}$$

Solution of equations (8) and (9) requires specifying ρ_e/ρ and l as functions of g plus the proper value of β.

Fay and Riddell (ref. 2) have numerically solved these equations for the axisymmetric stagnation point ($\beta = 0.5$) and a variety of representative flight conditions. Numerical solution of these equations for the two-dimensional case ($\beta = 1.0$) and various representative flight conditions has been made by Kemp, Rose, and Detra in reference 1 but has not been explicitly presented as the two-dimensional stagnation solution.

For the case of stagnation-point flow it can be shown that by using equations (4) to (7) and replacing u_e by

$$u_e = \left(\frac{du_e}{dx}\right)_{x=0} x \tag{12}$$

the heat-transfer parameter becomes:

Axisymmetric:

$$\left(\frac{Nu}{\sqrt{Re_w}}\right)_{as} = \sqrt{2}\left(\frac{g_\eta}{1 - g_w}\right)_{\substack{\eta=0\\ \beta=0.5}} \tag{13}$$

Two dimensional:

$$\left(\frac{Nu}{\sqrt{Re_w}}\right)_{2D} = \left(\frac{g_\eta}{1 - g_w}\right)_{\substack{\eta=0\\ \beta=1.0}} \tag{14}$$

Plotted in figure 1 are the theoretical values of $Nu/\sqrt{Re_w}$ obtained by using the flight conditions listed in the table of reference 2. It is apparent that $Nu/\sqrt{Re_w}$ can be easily correlated in terms of the ratio of $\rho\mu$ at the edge of the boundary layer to that at the wall. For $\rho_0\mu_0/\rho_{wo}\mu_{wo}$ equal to 1.0, the solutions of reference 3 have been presented, and they too correlate well.

For flow at the stagnation point of an axisymmetric blunt body, reference 2 has shown that when $L_i = 1.0$ and $\overline{Pr} = 0.71$ the theoretical values of $Nu/\sqrt{Re_w}$ for a real fluid can be approximated by

$$\left(\frac{Nu}{\sqrt{Re_w}}\right)_{as} \cong 0.67\left(\frac{\rho_0\mu_0}{\rho_{wo}\mu_{wo}}\right)^{0.40} \tag{15}$$

For the case of two-dimensional stagnation-point flow and similar conditions, the results of reference 1 yield

$$\left(\frac{Nu}{\sqrt{Re_w}}\right)_{2D} \cong 0.495\left(\frac{\rho_0\mu_0}{\rho_{wo}\mu_{wo}}\right)^{0.42} \tag{16}$$

E-260

In reference 4, Beckwith has compared the real-gas axisymmetric stagnation-point solution of reference 2 with the perfect-gas solution. This comparison revealed that the appropriate perfect-gas solution is one for which the total variation of $\rho\mu$ across the boundary layer is the same as in the real-gas case. Reference 4 then concludes that the heat transfer at the stagnation point is not sensitive to the effects of dissociation on the density and specific heat within the boundary layer.

Beckwith further reasoned that the two-dimensional perfect-gas solution could be extended in the same manner. In fact, he found that the heat-transfer parameter $Nu/\sqrt{Re_w}$ at the stagnation line of a two-dimensional yawed or unyawed body could be correlated in terms of $\rho\mu$ provided the actual conditions in the external flow at the stagnation line are used to determine the proper $\rho\mu$ ratio.

The results of reference 4 at the stagnation line of a yawed or unyawed two-dimensional body can be represented as

$$\left(\frac{Nu}{\sqrt{Re_w}}\right)_{2D} \cong 0.5 \left(\frac{\rho_0\mu_0}{\rho_{wo}\mu_{wo}}\right)^{0.44} \tag{17}$$

This equation agrees very well with the real-gas solution (eq. (16)), thereby completely justifying the extrapolation of reference 4 to the two-dimensional stagnation line. For the case of a stagnation line on a yawed two-dimensional body, complete justification is not possible since no real-gas solutions are available for comparison; however, the extrapolation does appear very reasonable.

Although no real-gas solutions are available for a three-dimensional stagnation point, a reasonable estimate can be obtained from the results of Reshotko in reference 5. These results indicate that the heat-transfer parameter can be expressed as

$$\left(\frac{Nu}{\sqrt{Re_w}}\right)_{3D} = 0.5 \sqrt{1 + \left(\frac{R_x}{R_z}\right)^{1/2}} \left(\frac{\rho_0\mu_0}{\rho_{wo}\mu_{wo}}\right)^{0.44} \tag{18}$$

where R_x is the smaller principal radius of curvature and R_z is the larger principal radius of curvature. For $R_x/R_z = 0$, equation (18) reduces exactly to the two-dimensional expression (eq. (17)). However, for $R_x/R_z = 1$, equation (18) deviates only slightly from the axisymmetric case (eq. (15)).

Lewis Number Different From 1.0

The previous results have been for a boundary layer in thermodynamic equilibrium with a Lewis number of 1.0 and a Prandtl number of 0.71. If the Lewis number is not equal to 1.0, then from reference 2 the energy equation is written as

$$(l g_\eta/\mathrm{Pr})_\eta + f g_\eta + \left[\frac{l}{\mathrm{Pr}} \sum \frac{c_{i,e}}{H_e} (h_i - h_i^o)(L_i - 1) s_{i,\eta}\right]_\eta = 0 \qquad (19)$$

Numerical solution of equations (19) and (8) for the axisymmetric stagnation point and values of L_i from 1 to 2 are presented in reference 2. The results of reference 2 indicate that the effect of Lewis number on the heat-transfer parameter can be simply correlated and is best given by

$$\frac{\mathrm{Nu}/\sqrt{\mathrm{Re}_w}}{\left(\mathrm{Nu}/\sqrt{\mathrm{Re}_w}\right)_{L_i=1}} \cong 1 + \left(L_i^{0.52} - 1\right) \frac{h_D}{H_e} \qquad (20)$$

where the dissociation energy per unit mass of air h_D is defined as

$$h_D = \sum_{\mathrm{atoms}} c_{i,e}(-h_i^o) \qquad (21)$$

The value of h_D can be estimated by using appendix A. Although equation (20) is a stagnation point in an axisymmetric flow, extension to the stagnation point in a two- or three-dimensional flow would probably be accurate.

Frozen Flow

Since it is possible for frozen flow to exist (very slow reaction rates), it is worthwhile to include its effect on the heat-transfer parameter. At a Lewis number of 1.0, the frozen and equilibrium boundary-layer equations are identical, and there is no effect. For Lewis numbers other than 1.0 the results of reference 2 indicate that the effect of Lewis number on the frozen-flow heat transfer can be simply correlated and is best given by

$$\frac{\mathrm{Nu}/\sqrt{\mathrm{Re}_w}}{\left(\mathrm{Nu}/\sqrt{\mathrm{Re}_w}\right)_{L_i=1}} \cong 1 + \left(L_i^{0.63} - 1\right) \frac{h_D}{H_e} \qquad (22)$$

E-260

The effect of Prandtl number on the heat-transfer parameter has been considered in references 2 and 3. The same correction can be applied to the results presented herein and is included in the following expressions for the heating rates in the next section.

Engineering Calculation of Heat-Transfer Rate in Stagnation Region

Stagnation point of an axisymmetric blunt body $(n = 1)$. - The heat-transfer rate at the stagnation point of a blunt body for a Lewis number other than 1.0* and a Prandtl number of 0.71 using equations (7), (12), (15), (20), and (22) can be written as

$$q_0 = 0.94\left(\frac{\rho_0\mu_0}{\rho_{wo}\mu_{wo}}\right)^{0.4}\left[\rho_w\mu_w\left(\frac{du_e}{dx}\right)_{x=0}\right]^{1/2}\left[1 + (L_i^{\alpha} - 1)\frac{h_D}{H_e}\right](H_e - h_w) \tag{23}$$

where $\alpha = 0.52$ for thermodynamic equilibrium and 0.63 for frozen flow. In order to correct for a $\overline{Pr}$ other than 0.71, replace the constant 0.94 in equation (23) by $0.76\ \overline{Pr}^{-0.6}$.

For axisymmetric bodies which are not too blunt (see below) the velocity gradient $(du_e/dx)_{x=0}$ can be obtained from modified Newtonian flow and is expressed as

$$\left(\frac{du_e}{dx}\right)_{x=0} = \frac{1}{R_N}\sqrt{\frac{2(p_{wo} - p_1)}{\rho_0}} \tag{24}$$

For very blunt bodies experimental pressure distributions must be used to determine the stagnation-point velocity gradient. Boison and Curtiss in reference 6 have correlated the experimental stagnation-point velocity gradient measurements with a bluntness parameter based on body sonic-point coordinates for a range of shapes from concave to an equivalent hemisphere.

Presented in figure 2 are the experimental velocity gradient data of reference 6. These data are correlated against the bluntness parameter x_s/r_s and are compared with Newtonian theory at a Mach number of 4.76. From these data it is seen that Newtonian theory can be used to approximate $(du_e/dx)_{x=0}$ for bodies less blunt than those with $x_s/r_s \geq 0.35$. The Mach 4.76 curve can be used to obtain a close approximation of the stagnation velocity gradient for any shape at hypersonic speeds, neglecting real-gas effects.

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*Ref. 2 concludes that the best value of Lewis number is approximately 1.4.

Stagnation line on a wing or control surface including effects of yaw (n = 0). - From equations (7), (12), (17), (20), and (22), heating rate at the stagnation line of a blunt two-dimensional leading edge for a Lewis number other than 1.0 and a $\overline{Pr}$ of 0.71 is written as

$$q_0 \cong 0.706\left(\frac{\rho_0\mu_0}{\rho_{wo}\mu_{wo}}\right)^{0.44}\sqrt{\rho_w\mu_w\left(\frac{du_e}{dx}\right)_{x=0}}\left[1 + (L_i^{\alpha} - 1)\frac{h_D}{H_e}\right](H_e - h_w) \tag{25}$$

where $\alpha = 0.52$ for thermodynamic equilibrium and 0.63 for frozen flow. In order to correct for $\overline{Pr}$ other than 0.71, replace the constant 0.706 by $0.565\ \overline{Pr}^{-0.6}$.

The velocity gradient for cylindrical leading edges is given by equation (24). For leading edges having large radii of curvature or irregular physical shapes, experimental pressure measurements should be used to determine the velocity gradient.

For the stagnation line of a yawed wing or control surface with $L_i = 1.0$ and $\overline{Pr} = 0.71$, the heating rate is given by

$$q_{0,\Lambda} \cong 0.706\left(\frac{\rho_0\mu_0}{\rho_{wo}\mu_{wo}}\right)_{\Lambda}^{0.44}\sqrt{(\rho_{wo}\mu_{wo})_{\Lambda}\left(\frac{du_e}{dx}\right)_{\Lambda,x=0}}\left(H_{aw,\Lambda} - h_w\right) \tag{26}$$

where

$$H_{aw,\Lambda} = H_e - \left(\frac{v_{e,\Lambda}^2}{2}\right)\left(1 - \sqrt{\overline{Pr}}\right)$$

For a yawed circular cylinder the results of reference 7 presented in figure 3 are used to determine the chordwise velocity gradient $(du_e/dx)_{\Lambda,x=0}$. If the leading edge cannot be treated as a circular cylinder, experimental pressure data should be used to determine the correct velocity gradient.

The method for determining the various parameters required in addition to the velocity gradient is discussed in appendix B. It is very important that the value of $\rho\mu$ be determined by using the actual conditions in the external flow at the stagnation line of the cylinder.

Three-dimensional flow near the stagnation point of a blunt three-dimensional body. - By using equations (7), (12), (18), (20), and (22), the heating rate at the stagnation point for three-dimensional flow is written as

$$q_0 = 0.706\left[1 + \left(\frac{R_x}{R_z}\right)^{1/2}\right]^{1/2}\left(\frac{\rho_0\mu_0}{\rho_{wo}\mu_{wo}}\right)^{0.44}\left[\rho_w\mu_w\left(\frac{du_e}{dx}\right)_{x=0}\right]^{1/2}$$

$$\times\left[1 + (L_i^{\alpha} - 1)\frac{h_D}{H_e}\right](H_e - h_w) \tag{27}$$

where $\alpha = 0.52$ for thermodynamic equilibrium and 0.63 for frozen flow, and R_x/R_z is the ratio of the smaller to larger radius of curvature. In order to correct for $\overline{Pr}$ other than 0.71, replace the constant 0.706 by $0.565\ \overline{Pr}^{-0.6}$.

The velocity gradient for the three-dimensional stagnation point is obtained in the manner suggested in reference 5 and is given by

$$\left(\frac{du_e}{dx}\right)_{x=0} = \frac{1}{R_x}\sqrt{\frac{2(p_{wo} - p_1)}{\rho_0}} \tag{28}$$

where R_x is the smaller principal radius of curvature.

BOUNDARY-LAYER FLOW WITH FAVORABLE PRESSURE GRADIENT

Lewis Number of Unity

The boundary-layer equations for the flow away from the stagnation point of an axisymmetric blunt body in dissociated air have been treated in detail by Kemp, Rose, and Detra in reference 1. The solutions of reference 1 are based on the assumptions of local similarity, $L_i = 1$, and $L_i^T = 0$. The assumption of local similarity requires that the velocity and enthalpy profiles remain similar (functions of η only), at least for an appreciable distance along the body, thereby reducing the boundary-layer equations to ordinary differential equations. Of course, the assumption of local similarity will not be valid for the entire distance away from the stagnation point. For example, some calculations made in reference 1 indicate that on a hemisphere the assumption of local similarity breaks down quite rapidly at angular positions greater than 60°. Nevertheless, for most cases of interest the results presented here should be adequate.

If the previous assumptions are used, the boundary-layer equations from reference 1 are:

Momentum equation:

$$\left(l f_{\eta\eta}\right)_\eta + f f_{\eta\eta} + \beta\left(\frac{\rho_e}{\rho} - f_\eta^2\right) = 0 \tag{29}$$

Energy equation:

$$\left(l g_\eta/\mathrm{Pr}\right)_\eta + f g_\eta + \left(\frac{u_e^2}{H_e}\right)\left[\left(1 - \frac{1}{\mathrm{Pr}}\right) l f_\eta f_{\eta\eta}\right]_\eta = 0 \tag{30}$$

with the boundary conditions given in equation (11).

The difference between equations (29) and (30) and the stagnation-point equations is the inclusion of the dissipation factor u_e^2/H_e (zero at the stagnation point) and the evaluation of the fluid properties. The fluid properties ρ_e/ρ and $\rho\mu/\rho_e\mu_e$ depend on the static enthalpy h instead of the stagnation enthalpy g where the relation between g and h is given by

$$h/H_e = g - f_\eta^2 \frac{u_e^2}{2H_e} \tag{31}$$

For most practical cases reference 1 has found that neglecting the dissipation term u_e^2/H_e and assuming the fluid properties to depend on g only do not significantly modify the values of $g_{\eta,w}/(1 - g_w)$ for a given value of β.

Omission of the dissipation factor and evaluation of the fluid properties in terms of g only reduce equations (29) and (30) to the stagnation-point equations with the proper value of the pressure-gradient parameter β. Solution of these equations can be obtained from reference 1 where Kemp, Rose, and Detra found that $\left[g_\eta/(1 - g_w)\right]_{\eta=0}$ can be correlated in terms of the pressure-gradient parameter β and the stagnation value of the $\rho\mu$ ratio.

Values of $\left[g_\eta/(1 - g_w)\right]_{\eta=0}$ required for heat-transfer calculation (see below) are presented in figure 4 as a function of $\rho_O\mu_O/\rho_{wo}\mu_{wo}$. These values were obtained by fairing lines through the solutions of reference 1 for $\beta = 0$, 0.5, 1.0, and 2.0. Reference 1 also indicates that within the range $0.15 \le \dfrac{\rho_O\mu_O}{\rho_{wo}\mu_{wo}} \le 0.55$ the value of $\left[g_\eta/(1 - g_w)\right]_{\eta=0}$ can be approximated as

$$\left(\frac{g_\eta}{1 - g_w}\right)_{\eta=0} \cong 0.458(1 + 0.096\sqrt{\beta})\left(\frac{\rho_O\mu_O}{\rho_{wo}\mu_{wo}}\right)^{0.438} \tag{32}$$

The relationship between $\left[g_\eta/(1 - g_w)\right]_{\eta=0}$ and the heat-transfer parameter $Nu/\sqrt{Re_w}$ at any point on an axisymmetric or two-dimensional blunt body can be expressed as

$$\frac{Nu}{\sqrt{Re_w}} = \frac{r^n\sqrt{\rho_w\mu_w u_e x}}{\sqrt{2\xi}}\left(\frac{g_\eta}{1 - g_w}\right)_{\eta=0}\left(\frac{H_e - h_w}{h_{aw} - h_w}\right) \tag{33}$$

A more detailed discussion of the preceding analysis is given in reference 1.

Away from the stagnation line the effects of yaw have not been rigorously considered for the real-gas case. However, as a first approximation the following method of calculation is suggested. This method is based on the assumptions of local similarity, $h_w/H_e \ll 1$, and correlation of the values of $\left[g_\eta/(1 - g_w)\right]_{\eta=0}$ for a yawed two-dimensional body in the same manner as for an unyawed body, provided the $\rho\mu$ ratio is determined by using the actual external fluid properties on the yawed body. It is implied in the assumption of $h_w/H_e \ll 1$ that the values of $g_\eta/g_{\eta,0}$ are approximately independent of yaw. From reference 4 this is approximately true for $T_w/T_0 = 0$. These assumptions are completely untested, and one would expect them to break down at least as rapidly as the assumption of local similarity for an unyawed body. Assuming a correlation of $\left[g_\eta/(1 - g_w)\right]_{\eta=0}$ with the actual $\rho\mu$ ratio of a yawed body is justified as a first approximation since reference 4 has found equation (17) to correlate successfully on the stagnation line of a yawed body provided the external flow properties on the yawed body are used.

In the preceding paragraphs no attempt has been made to include the effect of a Lewis number other than 1.0 or a Prandtl number other than 0.71. However, for engineering purposes reference 1 found that a correction for Lewis number and Prandtl number applied at the stagnation point is adequate over the entire body.

Calculation of Heat-Transfer Rate Around a Blunt Body with Favorable Pressure Gradient

Flow around an axisymmetric blunt body $(n = 1)$. - The heat-transfer rate at any point around the body using equations (7) and (33) is given by

$$q = \frac{\rho_w\mu_w u_e r}{Pr\sqrt{2\xi}}\left(\frac{g_\eta}{1 - g_w}\right)_{\eta=0}(H_e - h_w) \tag{34}$$

where $[g_\eta/(1 - g_w)]_{\eta=0}$ can be obtained from figure 4 as a function of $\rho_0\mu_0/\rho_{wo}\mu_{wo}$ and the pressure-gradient parameter β. Within the range $0.15 \leq \frac{\rho_0\mu_0}{\rho_{wo}\mu_{wo}} \leq 0.55$ reference 1 indicates the following approximation:

$$\left(\frac{g_\eta}{1 - g_w}\right)_{\eta=0} \cong 0.458(1 + 0.096\sqrt{\beta})\left(\frac{\rho_0\mu_0}{\rho_{wo}\mu_{wo}}\right)^{0.438} \tag{35}$$

The pressure-gradient parameter β is defined as

$$\beta = 2\left(\frac{\xi}{u_e}\right)\left(\frac{du_e}{d\xi}\right) \tag{36}$$

or, for a very cold wall where $\rho_w = p_e/RT_w$, the pressure-gradient parameter can be written as

$$\beta = 2\left(\frac{du_e}{dx}\right)\frac{\int_0^x \left(\frac{p_e}{p_{wo}}\right)u_e r^2\,dx}{\left(\frac{p_e}{p_{wo}}\right)u_e^2 r^2} \tag{37}$$

For axisymmetric stagnation-point flow $\beta = 0.5$, and for two-dimensional flow $\beta = 1.0$. A typical distribution of the local pressure-gradient parameter on a hemisphere and a cylinder is shown in figure 5.

By using equations (12) and (34) the local heating rate to the stagnation-point heating rate can be written as

$$\left(\frac{q}{q_0}\right)_{as} = \frac{\rho_w\mu_w u_e r}{2\sqrt{\xi}\,\sqrt{\rho_{wo}\mu_{wo}}}\left[\left(\frac{du_e}{dx}\right)_{x=0}\right]^{-1/2}\left(\frac{g_\eta}{g_{\eta,0}}\right)_{\eta=0} \tag{38}$$

where the value of $(g_\eta/g_{\eta,0})_{\eta=0}$ is obtained from figure 4.

In the expression for $(q/q_0)_{as}$ no attempt has been made to include the effect of a Lewis number other than 1.0 or a Prandtl number other than 0.71. However, for engineering purposes reference 1 indicates that a correction for Lewis number and Prandtl number applied at the stagnation point should be adequate over the entire body.

At this point it would be interesting to compare equation (38) with the results given by Lees in reference 8. Since Lees assumes

$\rho_w \mu_w = \rho_e \mu_e$, a comparison can be easily made, and Lees' expression for $(q/q_0)_{as}$ becomes

$$q_{as}^{+} = (q/q_0)_{as} = \frac{\rho_w \mu_w u_e r}{2\sqrt{\xi}\ \sqrt{\rho_{wo}\mu_{wo}}}\left[\left(\frac{du_e}{dx}\right)_{x=0}\right]^{-1/2} \tag{39}$$

Comparing equations (38) and (39) gives

$$\frac{\text{Heat, eq. (38)}}{\text{Heat, eq. (39)}} = \left(\frac{g_\eta}{g_{\eta,0}}\right)_w \tag{40}$$

The ratio of heating rate distribution presented in this report to that of Lees (eq. (40)) is compared in figure 6 for both the axisymmetric and two-dimensional case for various values of β and $\rho_0\mu_0/\rho_{wo}\mu_{wo}$. It is apparent from figure 6 that there is little error in Lees' method for the range of $\rho_0\mu_0/\rho_{wo}\mu_{wo}$ that will be encountered in practical cases.

However, as pointed out in references 1 and 2, the results of Lees do yield some error at the stagnation point. This fact is demonstrated in figure 1 where the present stagnation-point results are compared with reference 8. Nevertheless, Lees' method is satisfactory for engineering computations at the stagnation point.

Boundary-layer flow around leading edge of a wing or control surface including effects of yaw $(n = 0)$. - The heat-transfer rate at any point around an unyawed blunt leading edge ($L_i = 1.0$, $Pr = 0.71$) is given by

$$q = \frac{\rho_w \mu_w u_e}{\sqrt{2\xi}\ \overline{Pr}}\left(\frac{g_\eta}{1 - g_w}\right)_{\eta=0}(H_e - h_w) \tag{41}$$

where $\left[g_\eta/(1 - g_w)\right]_{\eta=0}$ can be obtained from figure 4 as a function of $\rho_0\mu_0/\rho_{wo}\mu_{wo}$ and the pressure-gradient parameter β. Within the range $0.15 \leq \dfrac{\rho_0\mu_0}{\rho_{wo}\mu_{wo}} \leq 0.55$ reference 1 indicates that $\left[g_\eta/(1 - g_w)\right]_{\eta=0}$ can be approximated by equation (35). The pressure-gradient parameter β is defined by equation (36) and for a cold wall (no dissociation) reduces to

$$\beta = 2\left(\frac{du_e}{dx}\right)\frac{\int_0^x \left(\frac{p_e}{p_{wo}}\right)u_e\ dx}{\left(\frac{p_e}{p_{wo}}\right)u_e^2} \tag{42}$$

since $k = 0$.

The ratio of the local heating rate to the heating rate at the stagnation point can be expressed as

$$q_{2D}^{+} = \left(\frac{q}{q_0}\right)_{2D} = \frac{\rho_w \mu_w u_e}{\sqrt{2\xi}\ \sqrt{\rho_{wo}\mu_{wo}}} \left[\left(\frac{du_e}{dx}\right)_{x=0}\right]^{-1/2} \left(\frac{g_\eta}{g_{\eta,0}}\right)_{\eta=0} \tag{43}$$

where $(g_\eta/g_{\eta,0})_{\eta=0}$ is obtained from figure 4. Although equation (43) is for $L_i = 1.0$ and $Pr = 0.71$, the corrections for Lewis number and Prandtl number made at the stagnation line should be adequate for the entire leading edge.

Since no real-gas solutions are available which have considered the effects of yaw away from the stagnation line, the following method is suggested. Assume that the values of $g_\eta/(1 - g_w)$ for a yawed surface can be correlated in the same manner as for an unyawed surface. Therefore, the ratio of heating rate at any point on the body to the stagnation line of the yawed surface yields

$$\frac{q_\Lambda}{q_{0,\Lambda}} \quad \frac{(\rho_w \mu_w)_\Lambda u_{e,\Lambda}}{\sqrt{2\xi}\ \sqrt{(\rho_{wo}\mu_{wo})_\Lambda}} \left[\left(\frac{du_e}{dx}\right)_{\Lambda,x=0}\right]^{-1/2} \left(\frac{g_\eta}{g_{\eta,0}}\right)_{\Lambda,\eta=0} \tag{44}$$

where $(du_e/dx)_{\Lambda,x=0}$ is the chordwise line velocity gradient (see fig. 3), $\xi = \int_0^x (\rho_w \mu_w)_\Lambda u_{e,\Lambda}\, dx$, and the values of $(g_\eta/g_{\eta,0})_{\Lambda,\eta=0}$ are obtained from figure 4 using the value of β and $(\rho_0\mu_0/\rho_{wo}\mu_{wo})_\Lambda$ on the yawed surface.

Once the pressure distribution is known around the yawed body, the external flow properties can be found. According to reference 7, the chordwise pressure distribution over a yawed circular cylinder can be reasonably represented by

$$\frac{p_{w,\Lambda} - p_1}{p_{wo,\Lambda} - p_1} = \cos^2\left(\frac{x}{R_N}\right) \tag{45}$$

For a noncircular yawed cylinder it would appear that a modified Newtonian pressure distribution would be adequate to determine the heating rates provided the correct chordwise velocity gradient is used at the stagnation line. A further discussion of the flow properties on a yawed surface is presented in appendix B.

FLOW WITH ZERO PRESSURE GRADIENT

Solutions for Case of a Dissociated Free Stream

Solutions for the laminar boundary layer with zero pressure gradient including the case of a compressible dissociated free stream have been presented in reference 9. A comparison of the reference enthalpy method (ref. 10) with these exact solutions indicates agreement to within 6 percent. Therefore, the reference enthalpy method can be recommended for calculating the heating rates on zero yaw cones, cylinders, wedges, and flat plates, for any flight conditions. However, the results presented in this report do not consider the viscous-inviscid interaction phenomena at the leading edge of wedges or flat plates, and, hence, the heat-transfer results do not apply in the vicinity of the leading edge.

The only yawed bodies on which a zero pressure gradient is possible are the flat plate, flat plate at angle of attack, and the wedge. The flat plate has been treated in reference 11 where it has been shown that the boundary layer is unaffected by yaw and the normal boundary-layer solutions apply in the planes containing the resultant stream velocity vector. The yawed wedge and the yawed flat plate at angle of attack, however, will require further discussion since they are three-dimensional problems.

Engineering Calculation of Heat-Transfer Rate for Zero-Pressure-Gradient Flow

Flow on zero-pressure-gradient portion of a body. - On the cylindrical or conical portion of the body, the heating rates can be calculated for all flight conditions by simply using a reference enthalpy method such as reference 10.

The heating rates on the cylindrical portion are identical to flat-plate results for the same wetted distance. For the cone, however, the heating rate is $\sqrt{3}$ times the flat-plate heating rate for the same wetted distance. This difference can be easily shown by using the Mangler transformation from the cone to the flat plate.

Flow on zero-pressure-gradient portion of a wing or control surface including effects of yaw**. - On most typical hypersonic vehicles the wing can be represented as a yawed flat plate, while the control surfaces can be considered yawed flat plates or wedges.

**This analysis does not apply in the vicinity of the leading edge where viscous-inviscid interaction phenomena occur.

Calculation of heating rates on a flat plate, a yawed flat plate, or an unyawed wedge is quite simple. A reference enthalpy method such as Eckert's (ref. 10) will yield good answers when the theory is applied along the streamwise coordinate.

However, the yawed-wedge-type control surface and the yawed flat plate wing or control surface at angle of attack (windward side only) cannot be treated exactly as in the preceding manner. Since they form three-dimensional boundary-layer problems, the "independence principle" (see ref. 11) indicates that the transverse momentum equation can be ignored and that a two-dimensional boundary-layer solution such as that of reference 10 should be applied along the coordinate normal to the leading edge.

Consider the flow through the shock of a yawed wedge or flat plate at angle of attack. In figures 7(a) and (b) the flow has been resolved into components parallel and normal to the shock. A coordinate system is set up such that x_n is the distance normal to the leading edge, $u_{e,\Lambda}$ is the component of velocity normal to the leading edge, and $v_{e,\Lambda}$ is the spanwise velocity component. Now, in order to use the method of reference 10 the inviscid flow properties $p_{w,\Lambda}$ and $u_{e,\Lambda}$ must be determined.

Using figures 7(a) and (b) and reference 12 gives the following expressions for $u_{e,\Lambda}$ and $p_{w,\Lambda}$:

Yawed wedge:

$$u_{e,\Lambda} = \sqrt{u_2^2 + w_2^2} \tag{46}$$

$$u_2 = V_1 \cos \Lambda \cos(90^\circ - \theta) \frac{\tan(\theta - \delta)}{\tan \theta} \tag{47}$$

$$w_2 = V_1 \cos \Lambda \sin(90^\circ - \theta) \tag{48}$$

$$\frac{p_{w,\Lambda}}{p_1} = \left(1 - \frac{u_2}{u_1}\right)\gamma_1 M_1^2 \cos^2\Lambda \sin^2\theta + 1 \tag{49}$$

$$\theta = \theta(\delta, V_1 \cos \Lambda), \text{ see fig. 3 of ref. 12} \tag{50}$$

Notice that both θ and δ are measured with reference to the yawed flight velocity $V_1 \cos \Lambda$. The wedge angle δ should not be confused with the root angle α. However, if the root angle is specified, then

$$\sin \delta = \sin \alpha \cos \Lambda \tag{51}$$

E-260

Yawed flat plate at angle of attack:

$$u_{e,\Lambda} = \sqrt{u_2^2 + w_2^2} \tag{46}$$

$$u_2 = V_{eff} \cos(90^\circ - \theta) \frac{\tan(\theta - \alpha')}{\tan \theta} \tag{52}$$

$$w_2 = V_{eff} \sin(90^\circ - \theta) \tag{53}$$

$$\frac{p_{w,\Lambda}}{p_1} = \left(1 - \frac{u_2}{u_1}\right)\gamma_1 M_{eff}^2 \sin^2\theta + 1 \tag{54}$$

$$\theta = \theta(\alpha', V_{eff}), \text{ see fig. 3 of ref. 13}$$

$$M_{eff} = \frac{V_{eff}}{a_1} \tag{55}$$

$$V_{eff} = V_1 \sqrt{\sin^2\alpha + \cos^2\alpha \cos^2\Lambda} \tag{56}$$

where α is the angle of attack of the vehicle and α' the effective flow deflection angle given by

$$\cos \alpha' = \frac{\cos \alpha \cos \Lambda}{\sqrt{1 - \sin^2\alpha \cos^2\Lambda}} \tag{57}$$

Therefore, by writing the expression for the heating rate of reference 10 in terms of the flow properties along the coordinate normal to the leading edge, the following is obtained:

$$q = 0.332 \frac{\rho^* u_{e,\Lambda}}{\sqrt{Re_{x,n}^*}} (Pr^*)^{-2/3}(h_{aw} - h_w) \tag{58}$$

where $u_{e,\Lambda}$ is the velocity along x_n, Pr^* is the Prandtl number based on the reference temperature, and the following expressions hold true:

$$\left.\begin{aligned} \rho^* &= \frac{p_{w,\Lambda} m_o}{Z^* R T^*} \\ Re_{x,n}^* &= \frac{\rho^* u_{e,\Lambda} x_n}{\mu^*} \\ \mu^* &= \mu^*(T^*) \\ T^* &= T^*(p_{w,\Lambda}, h^*) \end{aligned}\right\} \tag{59}$$

$$\left.\begin{aligned}
h^* &= h_{e,\Lambda} + 0.5(h_w - h_{e,\Lambda}) + 0.22(h_{aw} - h_{e,\Lambda}) \\
h_{e,\Lambda} &= H_{e,\Lambda} - \frac{u_{e,\Lambda}^2}{2} \\
H_{e,\Lambda} &= H_e - \frac{v_{e,\Lambda}^2}{2} \\
h_{aw} &= H_e - \frac{v_{e,\Lambda}^2 + u_{e,\Lambda}^2}{2}\left(1 - \sqrt{Pr^*}\right) \\
Pr^* &= Pr^*(T^*)
\end{aligned}\right\} \qquad (60)$$

Once $p_{w,\Lambda}$ is determined, the charts of reference 13 are helpful in determining some of the preceding quantities.

CONCLUDING REMARKS

A summary of some of the available laminar theories for computing heating rates on hypersonic vehicles has been presented. No attempt has been made to be overly rigorous in the presentation of these results. The engineering equations presented here, however, should enable one to predict adequately the laminar heating rates encountered on hypersonic vehicles.

Lewis Research Center
National Aeronautics and Space Administration
Cleveland, Ohio, September 21, 1959

E-260

E-260

APPENDIX A

FRACTION OF ENERGY IN DISSOCIATION

The fraction of energy in dissociation h_D/H_e can be estimated by using the results presented in reference 14. The stagnation enthalpy H_e includes the energy of dissociation h_D in addition to the kinetic energy. Therefore, the stagnation enthalpy can be expressed as

$$H_e = h_o + h_D + \frac{u_e^2}{2} = h_e + \frac{u_e^2}{2} \tag{A1}$$

and

$$h_e = h_o + h_D \tag{A2}$$

The enthalpy of the undissociated gas per unit mass can be approximated as

$$h_o \approx \bar{c}_p T_e \tag{A3}$$

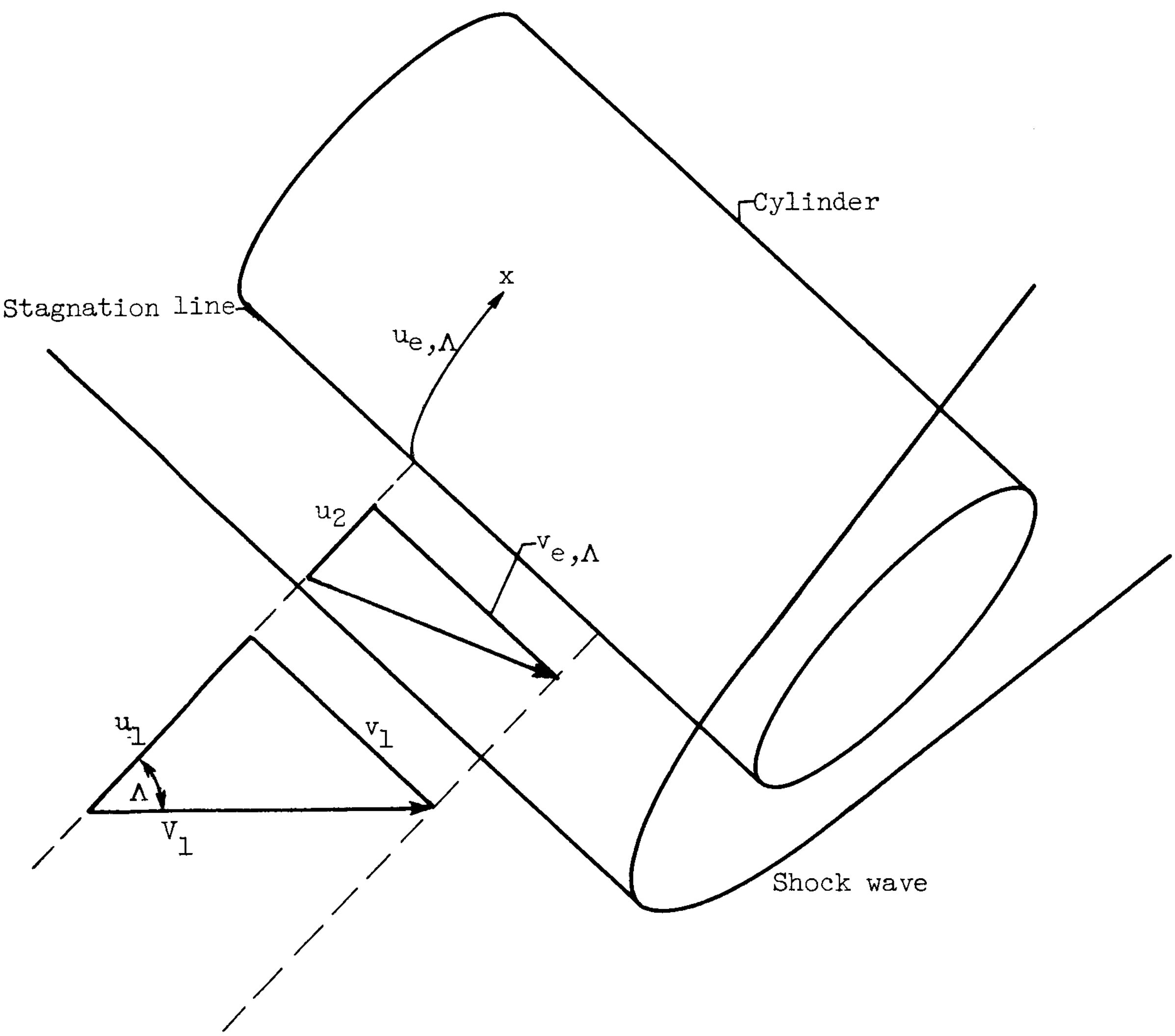
where $\bar{c}_p$ is the specific heat of the mixture. The fraction energy in dissociation can then be approximated as

$$\frac{h_D}{H_e} \approx \frac{h_e}{H_e} - \frac{\bar{c}_p T_e}{H_e} \tag{A4}$$

APPENDIX B

FLOW PROPERTIES ON A YAWED CYLINDER

In order to calculate the heating rates at the stagnation line by using equation (26), the values of $v_{e,\Lambda}$, $(\rho_{wo}\mu_{wo})_{\Lambda}$, and $(\rho_{o}\mu_{o})_{\Lambda}$ must be known. The coordinate system used to find the flow properties is defined by the following sketch:



E-260

where the x-coordinate is the distance along the cylinder surface measured in the chordwise direction from the leading edge. From the sketch it is easily seen that the spanwise velocity is represented as

$$v_1 = v_{e,\Lambda} = V_1 \sin \Lambda \tag{B1}$$

regardless of whether or not the fluid behaves as a perfect or real gas.

If the perfect-gas case is considered first, the determination of $(\rho_{wo}\mu_{wo})_\Lambda$ and $(\rho_0\mu_0)_\Lambda$ requires a knowledge of the pressure and temperature at the wall and in the external stream of the stagnation line. For this case the results of reference 7 can be used.

The absolute viscosity μ can be computed using the wall temperature and the external stream temperature $T_{0,\Lambda}$ where, from reference 7,

$$\frac{T_0}{T_{0,\Lambda}} = \frac{1 + \frac{\gamma_1 - 1}{2} M_1^2}{1 + \frac{\gamma_1 - 1}{2} M_1^2 \cos^2\Lambda} \tag{B2}$$

Since the density varies also with the pressure according to the equation of state, the static pressure must be determined from the inviscid flow. When the chordwise component of the free-stream velocity is supersonic, the wall pressure at the stagnation line can be written as

$$\frac{p_{wo,\Lambda}}{p_1} = \left(\frac{\gamma_1 + 1}{2} M_1^2 \cos^2\Lambda\right)^{\frac{\gamma_1}{\gamma_1-1}} \left[\frac{\gamma_1 + 1}{2\gamma_1 M_1^2 \cos^2\Lambda - (\gamma - 1)}\right]^{\frac{1}{\gamma_1-1}} \tag{B3}$$

For subsonic chordwise flow,

$$\frac{p_{wo,\Lambda}}{p_1} = \left(1 + \frac{\gamma_1 - 1}{2} M_1^2 \cos^2\Lambda\right)^{\frac{\gamma_1}{\gamma_1-1}} \tag{B4}$$

For a real gas in thermodynamic equilibrium, the external stream temperature $T_{0,\Lambda}$, the molecular weight ratio $Z_{0,\Lambda}$, and the pressure $p_{wo,\Lambda}$ must be known to determine $\rho\mu$. In order to find the pressure $p_{wo,\Lambda}$, the following relations are required:

$$\frac{p_2}{p_1} = \gamma_1 M_1^2 \cos^2\Lambda\left(1 - \frac{u_2}{u_1}\right) + 1 \tag{B5}$$

and

$$\frac{h_2}{h_1} = \frac{\gamma_1 - 1}{2} M_1^2 \cos^2\Lambda\left[1 - \left(\frac{u_2}{u_1}\right)^2\right] + 1 \tag{B6}$$

where u_2/u_1 can be found by using figure 2 of reference 12 and the normal Mach number $M_1 \cos \Lambda$ (indicated as $M_1 \sin \theta$ in ref. 12). Once p_2 and h_2 are found, knowing the value of the external stream enthalpy $H_{e,\Lambda}$, the stagnation-line pressure $p_{wo,\Lambda}$ can be determined from the charts of reference 13 by assuming constant entropy.

The external stream enthalpy $H_{e,\Lambda}$ is expressed as

$$H_{e,\Lambda} = H_e - \frac{v_e^2}{2} \tag{B7}$$

By knowing $p_{wo,\Lambda}$ and $H_{e,\Lambda}$, the charts of reference 13 can be used to find $Z_{0,\Lambda}$ and $T_{0,\Lambda}$. The density is then expressed as

$$\rho_{0,\Lambda} = \frac{p_{wo,\Lambda} m_o}{Z_{0,\Lambda} R T_{0,\Lambda}}$$

and

$$\rho_w = \frac{p_{wo,\Lambda} m_o}{Z_{wo,\Lambda} R T_w}$$

Away from the stagnation line, equation (44) is used to calculate the heating rates. In order to use this expression, the values of $\rho_{w,\Lambda}$ and $u_{e,\Lambda}$ and the local value of β must be determined. All these flow parameters can be easily found once the pressure distribution is prescribed. By using modified Newtonian flow theory the pressure distribution is given by equation (45):

$$\frac{p_{w,\Lambda} - p_1}{p_{wo,\Lambda} - p_1} = \cos^2\varphi$$

The pressure-gradient parameter β is then defined as

$$\beta = 2\left(\frac{\xi}{u_{e,\Lambda}}\right)\left(\frac{du_{e,\Lambda}}{d\xi}\right) \tag{B9}$$

where

$$\xi = \int_0^x (\rho_w \mu_w)_\Lambda u_{e,\Lambda}\, dx$$

or for very cold walls

$$\beta = 2 \frac{du_{e,\Lambda}}{dx} \frac{\int_0^x \left(\frac{p_{w,\Lambda}}{p_{wo,\Lambda}}\right) u_{e,\Lambda}\, dx}{\left(\frac{p_{w,\Lambda}}{p_{wo,\Lambda}}\right) u_{e,\Lambda}^2} \tag{B10}$$

Once the pressure is known, the velocity $u_{e,\Lambda}$ and density $\rho_{w,\Lambda}$ can be found from the charts of reference 13 and the following expressions:

$$\left.\begin{aligned} \rho_{w,\Lambda} &= \frac{p_{w,\Lambda} m_o}{Z_{w,\Lambda} R T_w} \\ \frac{u_e^2}{2} &= H_{e,\Lambda} - h_{e,\Lambda} \end{aligned}\right\} \tag{B11}$$

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E-260

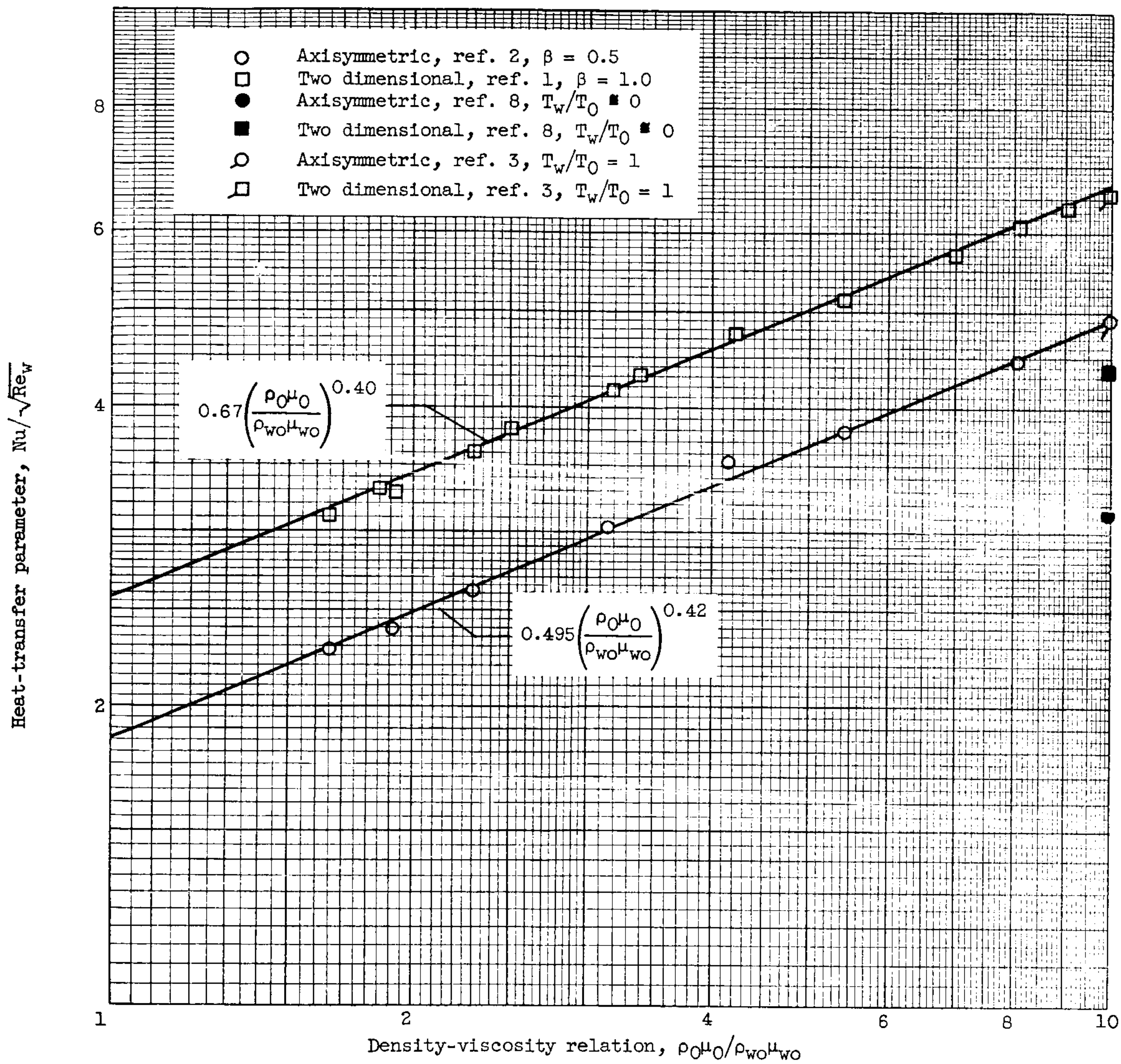


Figure 1. - Correlation of stagnation-point heat-transfer parameter.

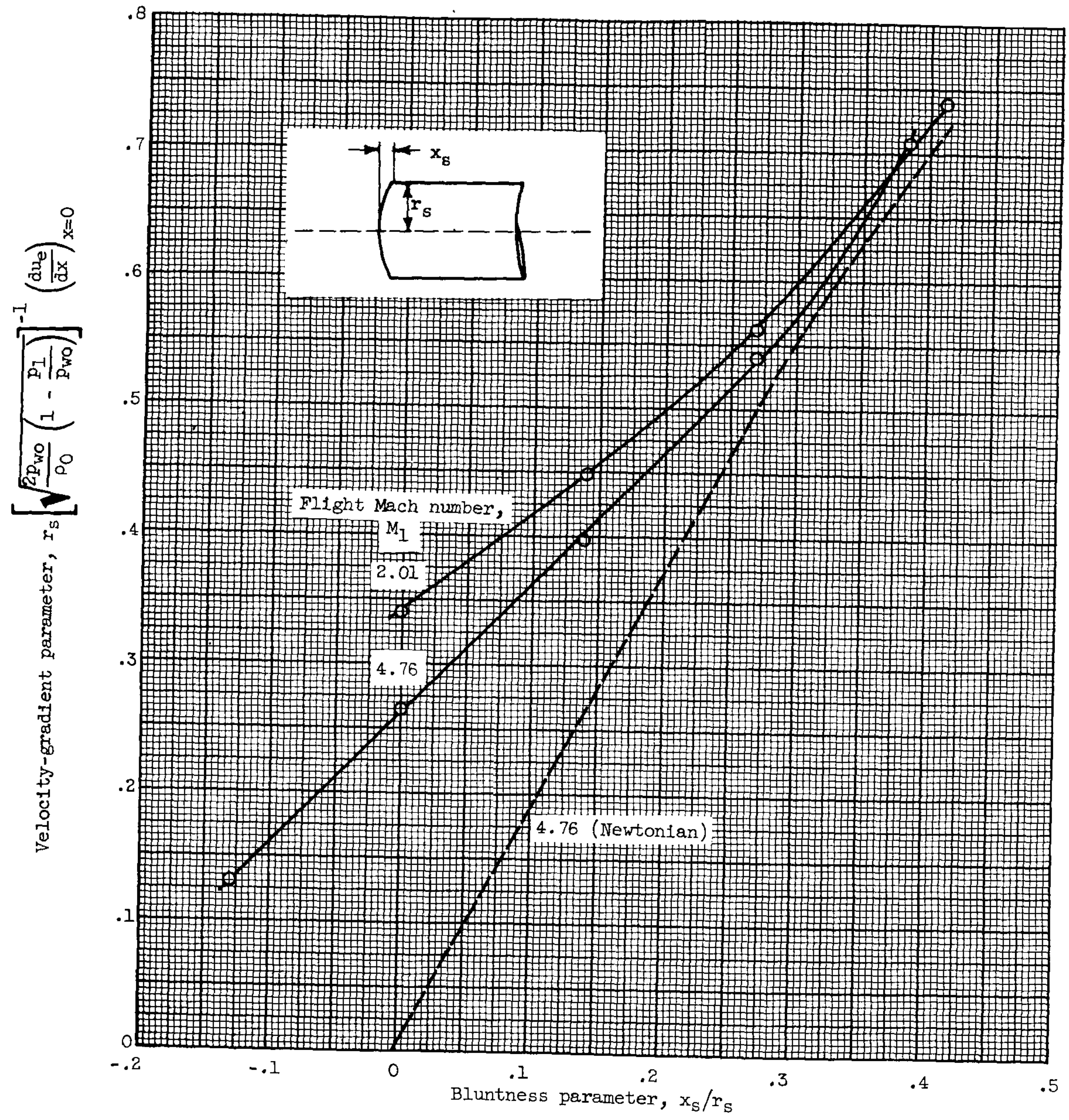


Figure 2. - Stagnation-point velocity gradient plotted against body bluntness. (Data from ref. 6.)

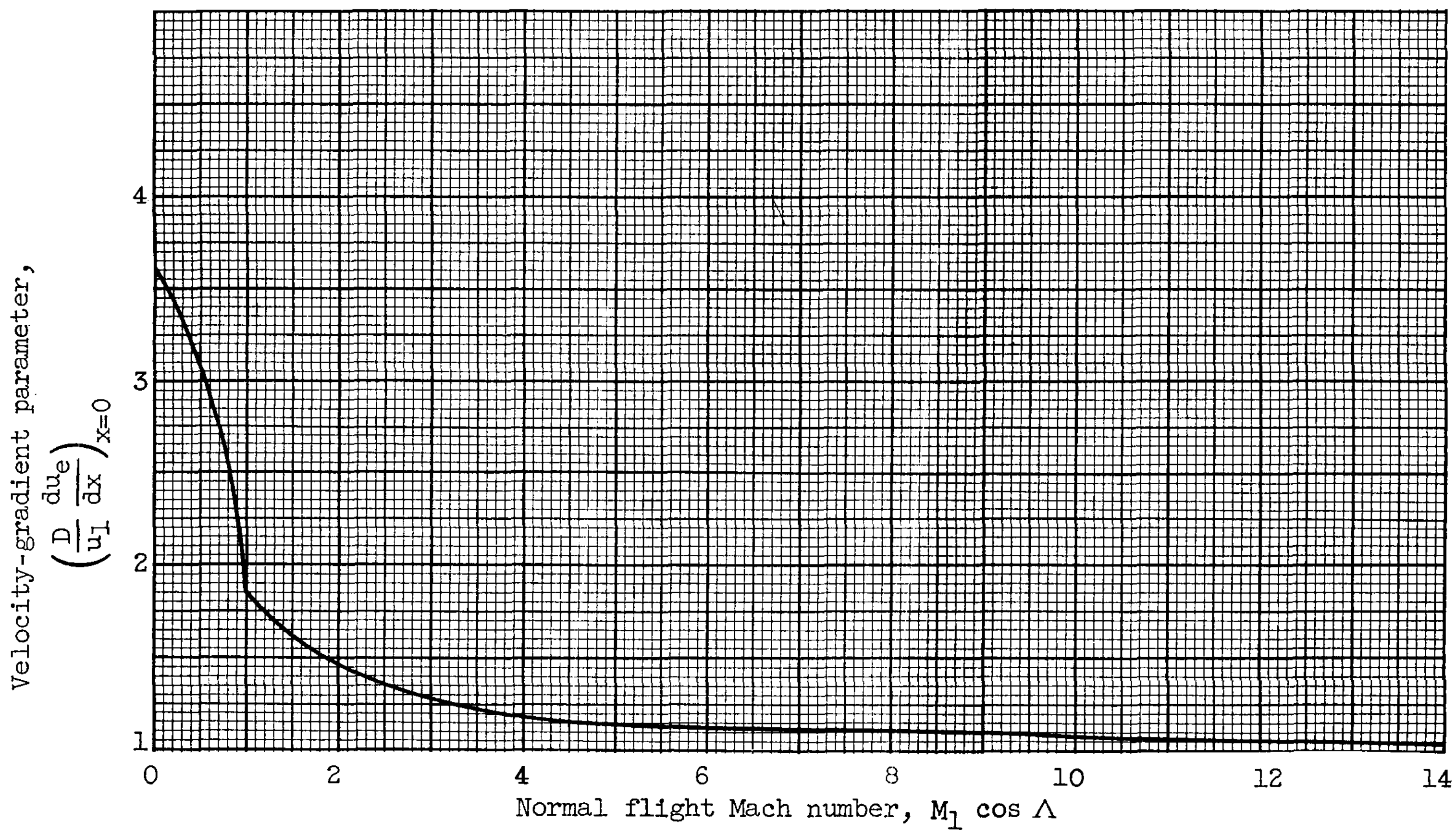


Figure 3. - Chordwise-velocity-gradient parameter as function of normal Mach number component for flow at stagnation line of yawed circular cylinder. Ratio of specific heats, 1.4. (Data from ref. 7.)

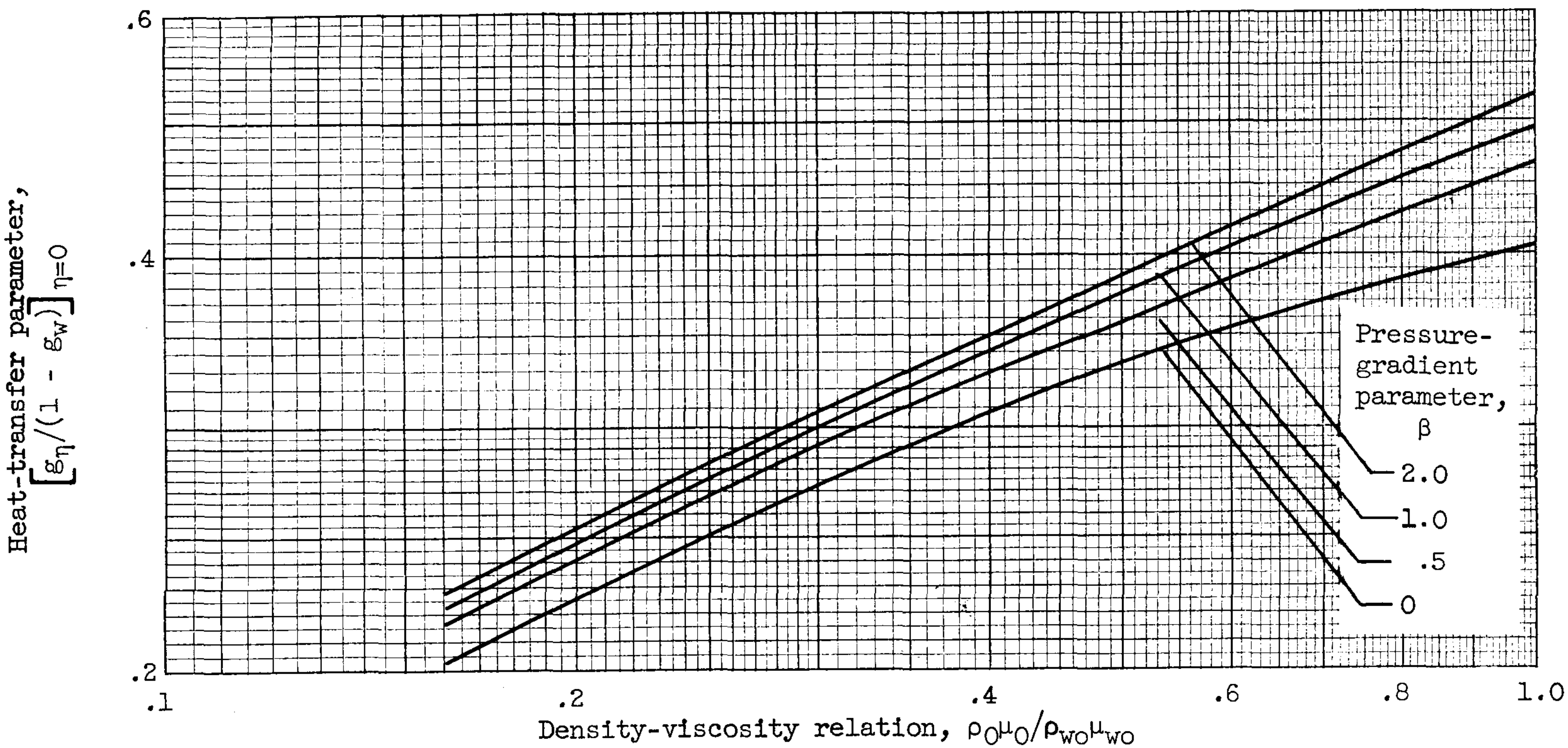


Figure 4. - Correlation of heat-transfer parameter.

E-260

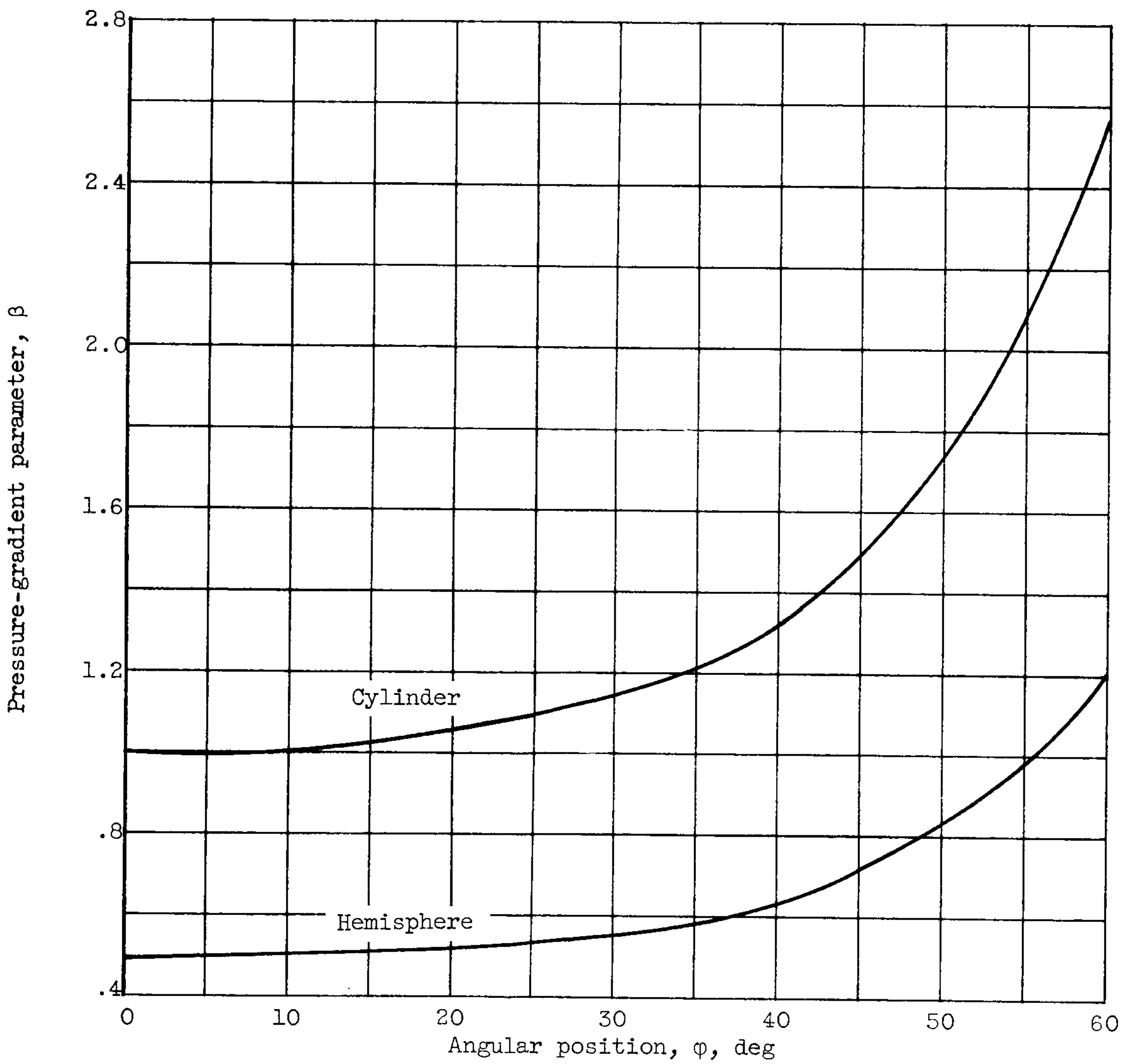


Figure 5. - Typical pressure-gradient-parameter distribution. Flight Mach number, >10.

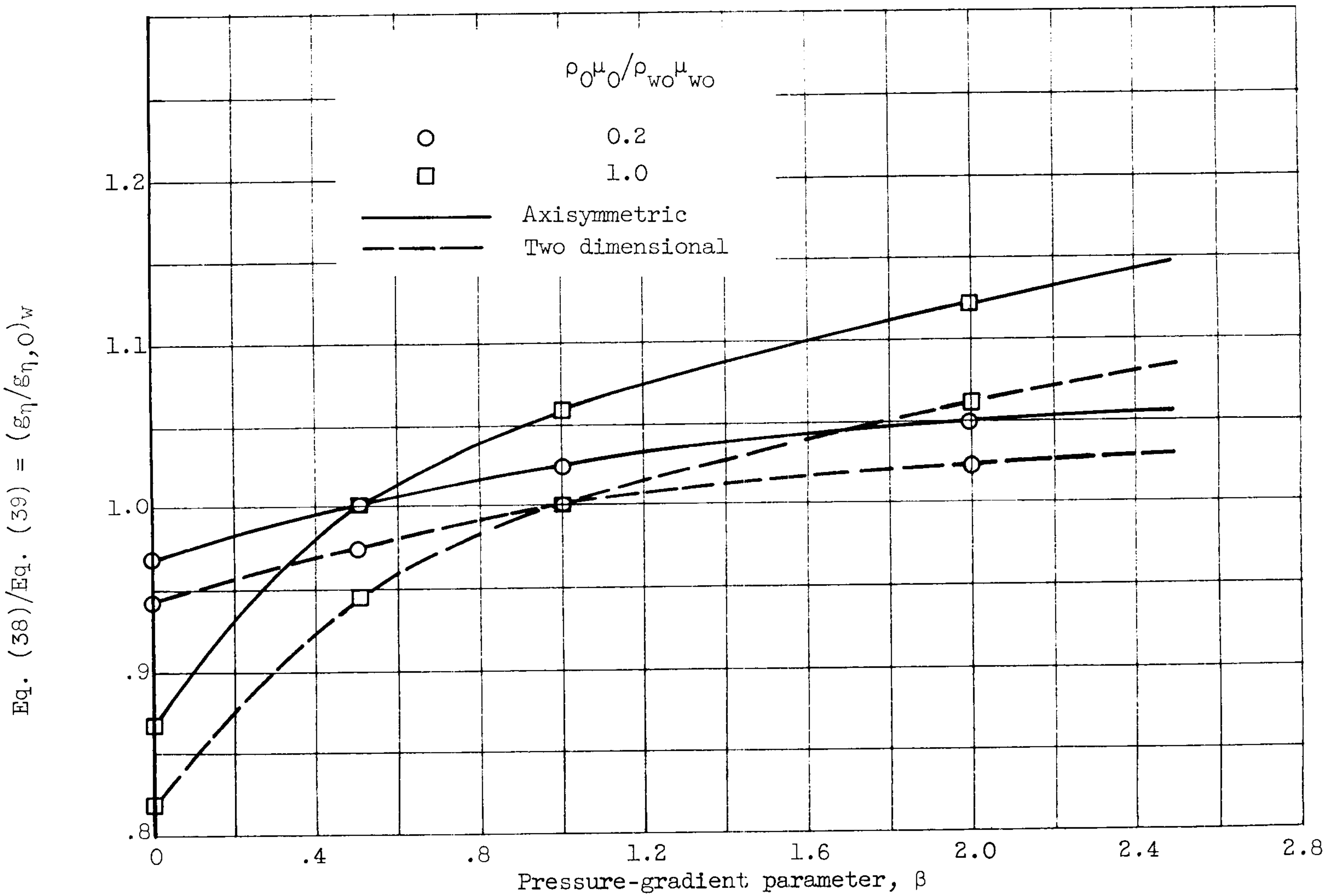
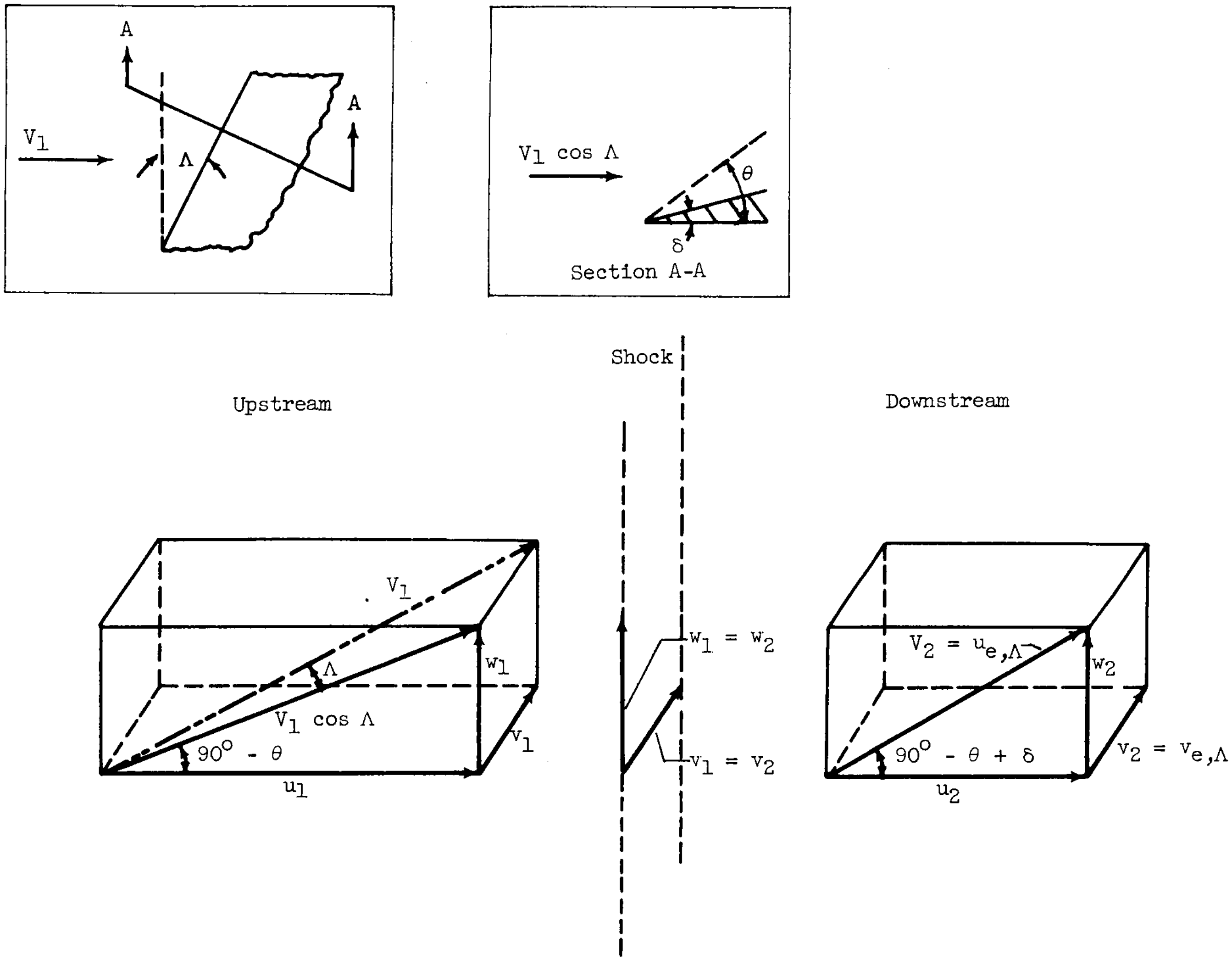
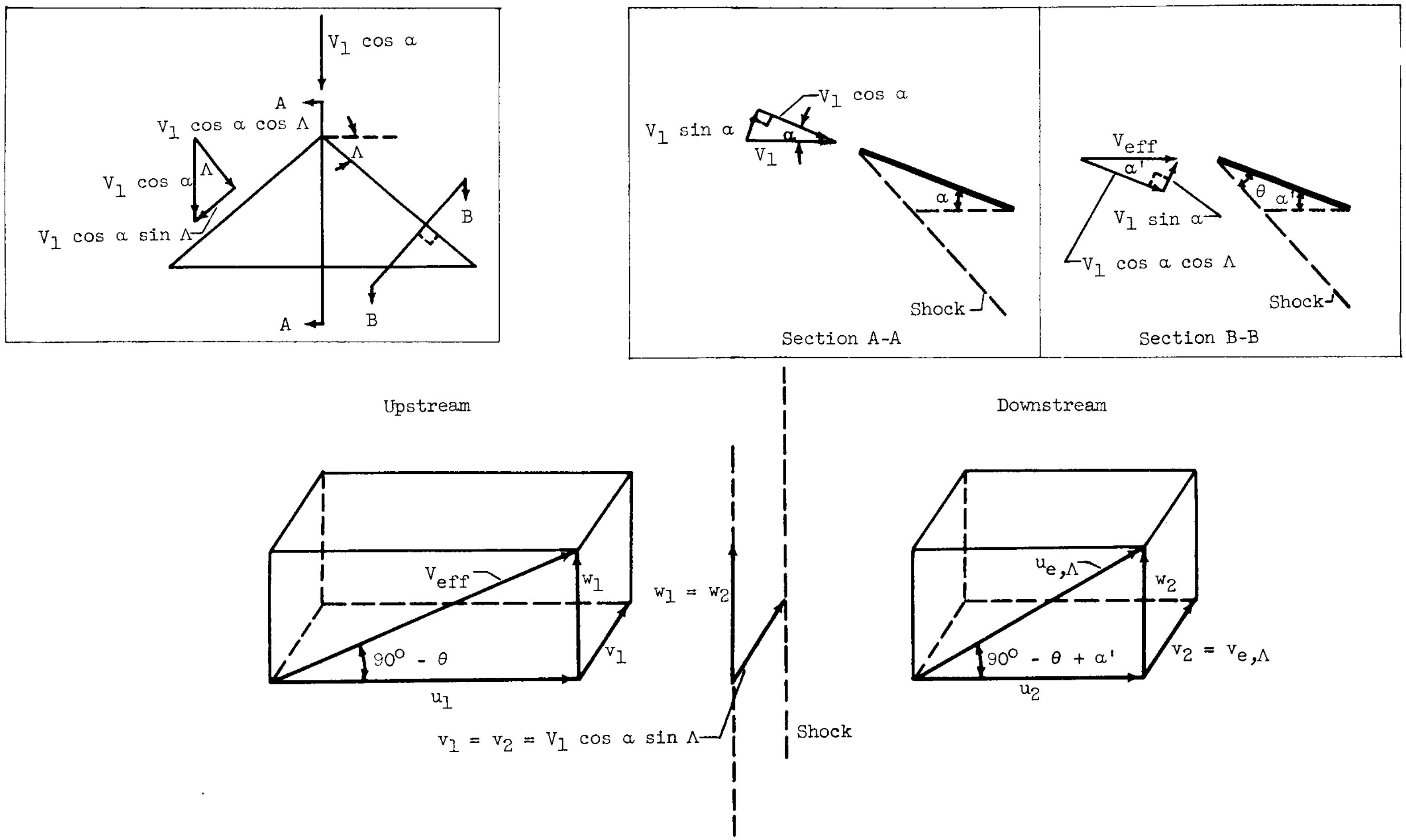


Figure 6. - Comparison of heat-transfer parameters as a function of β.



(a) Yawed wedge.

Figure 7. - Flow through an oblique shock.



(b) Flat plate at angle of attack (windward side).

Figure 7. - Concluded. Flow through an oblique shock.